UNIT #7 – POLYNOMIALS

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INTRODUCTION TO POLYNOMIALS
COMMON CORE ALGEBRA I

The way we write numbers in our systems is interesting because with only 10 digits, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, we are able to write whole numbers as large as we would like. This is because what we really are doing is counting how many powers of 10 that we have.

Exercise #1: Write each of the following numbers as a sum of multiples of powers of 10. The first is done as an example.

(a) 563 = 500 + 60 + 3
    = 5 \cdot 10^2 + 6 \cdot 10 + 3

(b) 274

(c) 3,842

(d) 5,081

(e) 21,478

We can now use algebra to replace the base of 10 with a generic base of $x$ (or whatever variable you like).

Exercise #2: Consider the number 63,735.

(a) As in #1, write this number as the sum of multiples of powers of 10.

(b) If $x = 10$, write this number in terms of an equivalent expression involving $x$.

The base of a polynomial certainly doesn't have to be 10. But, all polynomials have a form similar to your answer in letter (b). Let's define them a little more definitively.

POLYNOMIAL EXPRESSIONS

Any expression of the form: $ax^n + bx^{n-1} + cx^{n-2} + \ldots + \text{constant}$, where the exponents, $n, n-1, n-2$, etcetera are all positive integers. Note that not all powers need to be present because the coefficients, i.e. $a, b, c$, etcetera can be zero.

Exercise #3: Of the expressions shown below, circle all of them that represent polynomials. Discuss why the ones that aren't polynomials fail the definition above.

$4x^2 + 8x + 1$  
$9x^2 + 2x + \frac{1}{x}$  
$2^4 + 3^4 + 4^4$  
$2x^3 + 5x^3 - x + 8$
It is often important to place polynomials in their **standard form**. The standard form of a polynomial is simply achieved by writing it as an **equivalent expression** where the powers on the variables **always descend**.

**Exercise #4:** Write each of the following polynomials in standard form.

(a) $3x^2 + 5x^3 + 7 - 8x$

(b) $9x^4 + 2x - x^2 + 1$

(c) $3 - 2x - 5x^2$

$$5x^3 + 3x^2 - 8x + 7$$

$$9x^4 - x^3 + 2x + 1$$

$$-5x^2 - 2x + 3$$

Polynomials are simply abstract representations of numbers that we see every day and they behave like these numbers as well. Let's look at adding polynomials together.

**Exercise #5:** Consider the numbers 523 and 271.

(a) Write each as the sum of multiples of powers of 10 as previously done.

(b) Add these numbers by adding each individual power of 10.

(c) Use this idea to add: $5x^2 + 2x + 3$

$$+ 2x^2 + 7x + 1$$

$$\frac{7x^2 + 9x + 4}{\text{Finding sums of polynomials is fairly easy. Subtracting them, though, can lead to a lot of errors.}}$$

(d) Find the sum of the polynomials $-4x^2 + 9x - 3$

and $7x^2 - 5x + 4$.

$$\begin{array}{c}
-4x^2 \quad 9x \quad - 3 \\
+ 7x^2 \quad -5x \quad + 4
\end{array}$$

$$3x^2 + 4x + 1$$

**Exercise #6:** Find each of the following differences. Be careful and rewrite as an equivalent addition problem if necessary.

(a) $6x^2 + 5x + 3$

$$- 2x^2 - 4x + 7$$

$$\frac{4x^2 + 9x - 4}{\text{Exercise #7:} \text{ For each of the following, write an equivalent polynomial in simplest standard form.}}$$

(b) $(4x^2 - 2x + 7) - (2x^2 + x - 3)$

$$\frac{4x^2 + 2x + 7 + 2x^2 - x + 3}{6x^2 - 3x + 10}$$

**Exercise #7:** For each of the following, write an equivalent polynomial in simplest standard form.

(a) $6x^2 + 2x - 3x + 4x - 1$

$$5x^2 + 6x - 4$$

(b) $6x^2 + 2x - 3 - (x^2 + 4x - 1)$

$$5x^2 - 2x - 2$$

**COMMON CORE ALGEBRA I, UNIT #7 – POLYNOMIALS – LESSON #1**

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INTRODUCTION TO POLYNOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Write each of the following integers as multiples of powers of 10. The first is done as a reminder of this process.
   (a) 563
       \[563 = 500 + 60 + 3\]
       \[= 5 \cdot 10^2 + 6 \cdot 10 + 3\]
   (b) 278
   (c) 703
   (d) 5,378
   (f) 19,073

2. Consider the number 5,364.
   (a) Write this number as the sum of multiples of powers of 10 as in #1.
   (b) If \(x = 10\), write an expression in terms of \(x\) for the number 5,364.

3. Which of the following would be the value of the expression \(5x^2 + 2x^2 + 8x + 4\) when \(x = 10\) ?
   (1) 6,432
   (2) 2,854
   (3) 5,284
   (4) 528

4. Which of the following would be the value of the expression \(8x^2 + 2x + 3\) when \(x = 10\) ?
   (1) 823
   (2) 8,023
   (3) 8,203
   (4) 8,230

5. Which of the following is not a polynomial expression?
   (1) \(x^4\)
   (2) \(3^x\)
   (3) \(1 - 2x^3\)
   (4) \(6x + 1\)
6. Write each of the following polynomial expressions in standard form.

(a) \( 7x^3 + 4x^2 + 5 + 2x \) 
(b) \( 4 - x - 5x^3 \) 
(c) \( x^4 + x - 7x^2 + 2 \) 
(d) \( 2x + 1 - 3x^3 + 5x^2 \) 
(e) \( 4x^3 - 2x^2 + 6 - 8x \) 
(f) \( y^5 + y^{10} - y^2 + y^3 \) 

Find each of the following sums and differences. Write your answer in simplest standard form.

(a) \( 6x^2 - 2x + 8 + 3x^2 + 7x + 2 \) 
(b) \( x^2 + 4x^2 - 8x + 3 + 6x + 1 \) 
(c) \((5x^3 + 3x - 1) - (3x^2 - 6x + 4)\) 
(d) \((2x^3 - 5x^2 + 8x - 1) - (-4x^3 + 8x^2 - 3x - 9)\) 
(e) \(4x^2 + 6x + 5 - 2x^2 + 2x + 4\) 
(f) \((4x^3 + 6x - 3) - (3x^2 + 2x + 4)\) 

APPLICATIONS

8. A box has a width that is 2 inches greater than its height and a length that is 6 inches greater than its height. It's volume is given by the polynomial expression \( x^3 + 8x^2 + 12x \), where \( x \) is the box's height. What is the box's volume, in cubic inches, if its height is 10 inches?

(1) 1,812 
(2) 1,920 
(3) 182 
(4) 2,180 

REASONING

9. Polynomial expressions act a lot like integers because the structure of polynomials is based on the structure of integers. Based on the statement below about integers, make a statement about polynomials.

Statement About Integers: An integer added to an integer gives an integer.

Statement About Polynomials: ________________________________
MULTIPLYING POLYNOMIALS
COMMON CORE ALGEBRA I

Polynomials, as we saw in the last lesson, behave a lot like integers (whole numbers including the negatives). We saw that just like integers, adding one polynomial to another polynomial results in a third polynomial. The same will occur with multiplying them. First, a review problem.

Exercise #1: Monomials are the simplest of polynomials. They consist of one term (terms are separated by addition and subtraction). Find the following products of monomials.

(a) \(5x^2 \cdot 2x^3\)
\[10x^5\]

(b) \(-3x \cdot -8x\)
\[24x^2\]

(c) \(\frac{1}{2}x^2y^5 \cdot \frac{3}{4}x^3y\)
\[\frac{3}{8}x^4y^6\]

We have also used the Distributive Property in previous lessons to multiply polynomials that are more complicated.

Exercise #2: Find each of the following products in simplest form by using the distributive property once or twice.

(a) \(2(x^2 - 3x - 1)\)
\[6x^2 - 2x\]

(b) \(x^2(4x^2 + 3)\)
\[4x^4 + 3x^2\]

(c) \(-2x^2y^3(2xy - 3x)\)
\[-4x^3y^4 + 10x^3y^3\]

(d) \((x + 2)(x - 6)\)
\[x^2 - 6x + 2x - 12\]

(e) \((2x + 7)(x + 3)\)
\[2x^2 + 6x + 7x + 21\]

(f) \((3x - 2)(5x - 1)\)
\[15x^2 - 3x - 10x + 2\]

\[15x^2 - 13x + 2\]

Never forget that as we do these manipulations we are using properties of equality to produce equivalent expressions.

Exercise #3: Consider the product of the two binomial polynomials \((x - 1)(x - 3)\).

(a) Find this product and express it as a trinomial polynomial written in standard form. Fill in the result in the first row (third column) of table (b).

\[
(x - 1)(x - 3) \]
\[x^2 - 3x - x + 3\]
\[x^2 - 4x + 3\]

(b) Fill out the table below using TABLES on your calculator to show they are equivalent.

<table>
<thead>
<tr>
<th>x</th>
<th>((x - 1)(x - 3))</th>
<th>(x^2 - 4x + 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
We can evaluate more complicated products, just as we have done in the past with normal numbers. The key will always be the careful use of the *distributive property*.

**Exercise #4:** Find each of the following more challenging products.

(a) \((2x+5)^2\)

\[
(2x+5)(2x+5) = 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25
\]

(b) \((x+2)(x^2 + 4x + 3)\)

\[
x^2 + 4x + 3
\]

\[
\begin{array}{c|ccc}
   & x^2 & 4x & 3 \\
\hline
   x & 2x^2 & 8x & 6 \\
x^3 + 4x^2 + 3x + 2x^2 + 8x + 6 = x^3 + 6x^2 + 11x + 6
\end{array}
\]

(c) \((x-4)(x+3)(x-5)\)

\[
(x-4)(x^2 + 3x - 5x - 15) \\
= (x-4)(x^2 - 2x - 15)
\]

\[
x^3 - 2x^2 - 15x - 4x^2 + 8x + 60 = x^3 - 6x^2 - 7x + 60
\]

(d) \((3x+2)^3\)

\[
(3x+2)(3x+2)(3x+2)
\]

\[
(3x+2)(9x^2 + 6x + 4) \\
= 27x^3 + 54x^2 + 36x + 8
\]

**Exercise #5:** Consider the product \((3x+2)(2x+1)\).

(a) Write this product as an equivalent trinomial expression in standard form.

\[
(3x+2)(2x+1) = 6x^2 + 3x + 4x + 2 = 6x^2 + 7x + 2
\]

(b) How can you use your answer from (a) to evaluate the product \((32)(21)\)? Find the product and check using your calculator.
MULTIPLYING POLYNOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Write the following products as polynomials in either \( x \) or \( t \). The first is done as an example for you.

(a) \( 5x(2x-4) \)
\[
=(5x)(2x)-(5x)(4)
\]
\[
=10x^2-20x
\]
(b) \( 3(t+7) \)
\[
=3t+21
\]
(c) \( -4x(5x+1) \)
\[
=-20x^2-4
\]
(d) \( 4(t^2-3t+2) \)
\[
=4t^2-12t+8
\]
(e) \( x(x^2-2x-3) \)
\[
=x^3-2x^2-3x
\]
(f) \( -5(2t^2+3t-7) \)
\[
=-10t^2-15t^2+35t
\]

2. Perhaps the most important type of polynomial multiplication is that of two binomials. Make sure you are fluent with this skill. Write each of the following products as an equivalent polynomial written in standard form. The first problem is done as an example using repeated distribution.

(a) \( (x+5)(x-3) \)
\[
=(x+5)(x)+(x+5)(-3)
\]
\[
=x^2+5x-3x-15
\]
\[
=x^2+2x-15
\]
(b) \( (x-10)(x-4) \)
\[
=x^2-14x+40
\]
(c) \( (x+3)(x+12) \)
\[
=x^2+12x+3x+36
\]
\[
=x^2+15x+36
\]
(d) \( (2x+3)(5x+8) \)
\[
=10x^2+16x+15x+24
\]
\[
=10x^2+31x+24
\]
(e) \( (x-1)(x+2) \)
\[
=x^2+2x-x-2
\]
\[
=x^2-7x-2
\]
(f) \( (6x-5)(4x-3) \)
\[
=24x^2-18x-20x+15
\]
\[
=24x^2-38x+15
\]

3. Never forget that squaring a binomial also a process of repeated distribution. Write each of the following perfect squares as trinomials in standard form.

(a) \( (x+3)^2 \)
\[
=x^2+6x+9
\]
\[
=x^2+6x+9
\]
(b) \( (x-10)^2 \)
\[
=x^2-20x+100
\]
\[
=x^2-10x-100
\]
(c) \( (2x+3)^2 \)
\[
=4x^2+12x+9
\]
\[
=4x^2+12+9
\]
4. An interesting thing happens when you multiply two **conjugate binomials**. Conjugates have the property of having the same **terms** but differ by the operation between the two terms (in one case addition and in one case subtraction). Multiply each of the following **conjugate pairs** and state your answers in **standard form**. The first is done as an example

(a) \((x+3)(x-3)\)
\[= x(x-3) + 3(x-3)\]
\[= x^2 - 3x + 3x - 9\]
\[= x^2 - 9\]

(b) \((x-5)(x+5)\)
\[= x^2 + 5x - 5x - 25\]
\[= x^2 - 25\]

(c) \((10+x)(10-x)\)
\[= 100 - 10x + 10x - x^2\]
\[= 100 - x^2\]

(d) \((2t+3)(2t-3)\)
\[= 4t^2 - 9\]

(e) \((5t+1)(5t-1)\)
\[= 25t^2 - 5t + 5t - 1\]
\[= 25t^2 - 1\]

(f) \((8-3t)(8+3t)\)
\[= 64 + 24t - 24t - 9t^2\]
\[= 64 - 9t^2\]

5. Write each of the following products in **standard polynomial form**.

(a) \((x+3)(x-2)(x-8)\)
\[= (x+3)(x^2 - 2x - 8x + 16)\]
\[= (x+3)(x^2 - 10x + 16)\]

(b) \((x+2)(x-2)(x+3)(x-3)\) (Hint: try to use #4)
\[= (x^2 + 2x - 2x - 4)(x^2 - 3x - 9)\]
\[= (x^2 - 4)(x^2 - 9)\]

**REASONING**

6. Notice again how similar polynomials are to integers, i.e. the set \\{-3, -2, -1, 0, 1, 2, 3, \ldots\}. Write a statement below for polynomials based on the statement about integers.

**Statement About Integers:** An integer times an integer produces an integer.

**Statement About Polynomials:** A polynomial times a polynomial produces a polynomial.

7. Consider the product \((3x+1)^2\).

(a) Write this product in **standard trinomial form**.
\[= (3x+1)(3x+1)\]
\[= 9x^2 + 3x + 3x + 1\]
\[= 9x^2 + 6x + 1\]

(b) Use your answer in part (a) to determine the value of \(31^2\) without your calculator.
Factoring expressions is one of the gateway skills that is necessary for much of what we do in algebra for the rest of the course. The word factor has two meanings and both are important.

**THE TWO MEANINGS OF FACTOR**

1. **Factor (verb):** To rewrite an algebraic expression as an equivalent product.
2. **Factor (noun):** An algebraic expression that is one part of a larger factored expression.

**Exercise #1:** Consider the expression \(6x^2 + 15x\).

(a) Write the individual terms \(6x^2\) and \(15x\) as completely factored expressions. Determine their greatest common factor.

\[
6x^2 = 2x(3x) \quad 15x = 5(3x)
\]

(b) Using the Distributive Property, rewrite \(6x^2 + 15x\) as a product involving the gcf from (a).

\[
3x(2x + 5)
\]

(c) Evaluate both \(6x^2 + 15x\) and the factored expression you wrote in (b) for \(x = 2\). What do you find? What does this support about the two expressions?

It is important that you are fluent reversing the distributive property in order to factor out a common factor (most often the greatest common factor). Let's get some practice in the next exercise just identifying the greatest common factors.

**Exercise #2:** For each of the following sets of monomials, identify the greatest common factor of each. Write each term as an extended product (if necessary).

(a) \(12x^3\) and \(18x\)

\[
6x(2x^2 + 3)
\]

(b) \(5x^4\) and \(25x^2\)

\[
5x^2(x^2 + 5)
\]

(c) \(21x^2y^5\) and \(14xy^3\)

\[
7xy^3(3x + 2y^2)
\]

(d) \(24x^3, 16x^2, \) and \(8x\)

\[
8x(3x^2 + 2x + 1)
\]

(e) \(20x^3, -12x^2, \) and \(28x\)

\[
4x(5x^2 - 3x + 7)
\]

(f) \(18x^2y^7, 45x^3y, \) and \(90xy^3\)

\[
9xy(2xy + 5y + 10y^2)
\]
Once you can identify the greatest common factor of a set of monomials, you can then easily use it and the distributive property to write equivalent factored expressions.

Exercise #3: Write each polynomial below as a factored expression involving the greatest common factor of the polynomial.

(a) $6x^2 + 10x$
$2(x(3x+5))$

(b) $3x - 24$
$3(x-8)$

(c) $10x^2 - 15x$
$5(2x-3)$

(d) $4x^2 + 8x + 24$
$4((x^2+2x+6))$

(e) $6x^3 - 8x^2 + 2x$
$2x(3x^2 - 4x + 1)$

(f) $10x^3 - 35x^2$
$5x^2(2x-7)$

(g) $10x^3 - 40x - 50$
$10(x^2-4x-5)$

(h) $8x^4 - 2x^2$
$2x^2(4x^2 - 1)$

(i) $8x^3 + 24x^2 - 32x$
$8x(x^3 + 3x - 4)$

Being able to fluently factor out a gcf is an essential skill. Sometimes greatest common factors are more complicated than simple monomials. We have done this type of factoring back in Unit #1.

Exercise #4: Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor.

(a) $(x+5)(x-1) + (x+5)(2x-3)$
$(x+5)((x-1)+(2x-3))$

(b) $(2x-1)(2x+7) - (2x-1)(x-3)$
$(2x-1)((2x+7) - (x-3))$

$2x^2 + 20x - x - 10$
$2x^2 - 19x - 10$
**Factoring Polynomials**

**Common Core Algebra I Homework**

**Fluency**

1. Identify the greatest common factor for each of the following sets of monomials.
   - (a) \(6x^2\) and \(24x^3\)
   - (b) \(15x\) and \(10x^2\)
   - (c) \(2x^4\) and \(10x^2\)
   - (d) \(2x^3\), \(6x^2\), and \(12x\)
   - (e) \(16r^2\), \(48r\), and \(80\)
   - (f) \(8t^3\), \(12t^3\), and \(16t\)

   - (i) \(6\) \(2x^2\)
   - (j) \(2x\)
   - (k) \(10\)
   - (l) \(4\) \(1\)

2. Which of the following is the greatest common factor of the terms \(36x^2y^4\) and \(24xy^7\)?
   - (1) \(12xy^4\)
   - (2) \(24x^2y^7\)
   - (3) \(6x^2y^3\)
   - (4) \(3xy\)

3. Write each of the following as equivalent products of the polynomial's greatest common factor with another polynomial (of the same number of terms). The first is done as an example.
   - (a) \(8x - 28\)
     - \(= 4(2x - 7)\)
   - (b) \(50x + 30\)
     - \(= 10(5x + 3)\)
   - (c) \(24x^2 + 32x\)
     - \(= 8(3x + 4)\)
   - (d) \(18 - 12x\)
     - \(= 6x^3 + 12x^2 - 3x\)
   - (e) \(3x(3 - 2x)\)
     - \(= 3x(2x^2 + 4x - 1)\)
   - (f) \(x^2 - x\)
     - \(= x(x - 1)\)
   - (g) \(10x^2 + 35x - 20\)
     - \(= 5(2x^2 + 7x - 4)\)
   - (h) \(21x^3 - 14x\)
     - \(= 7x(3x^2 - 2)\)
   - (i) \(36x - 8x^2\)
     - \(= 4x(9 - 2x)\)
   - (j) \(30x^3 - 75x^2\)
     - \(= 15x^2(2x - 5)\)
   - (k) \(-16t^2 + 96t\)
     - \(-16(t^2 - 6t)\)
   - (l) \(4t^3 - 32t^2 + 12t\)
     - \(= 4t(t^2 - 8t + 3)\)

4. Which of the following is not a correct factorization of the binomial \(10x^2 + 40x\)?
   - (1) \(10x(x + 4)\)
   - (2) \(10(x^2 + 4)\)
   - (3) \(5x(2x + 4)\)
   - (4) \(5x(2x + 8)\)
5. Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor. Watch out for the subtraction problems (b) and (d).

(a) \( (x+5)(x+1)+(x+5)(x+8) \)
\( (x+5)(x+1+x+8) \)
\( (x+5)(2x+9) \)
\( 2x^2+14x+45 \)

(b) \( (2x-1)(3x+5)-(2x-1)(x+4) \)
\( (2x-1)(3x+5-x-4) \)
\( (2x-1)(2x+1) \)
\( 4x^2+2x-3x-1 \)
\( 4x^2-1 \)

(c) \( (x-7)(x-9)+(x-7)(4x+5) \)
\( (x-7)(x-9+4x+5) \)
\( (x-7)(5x-4) \)
\( 5x^2-4x-35x+28 \)
\( 5x^2-39x+28 \)

Applications

6. The area of a rectangle is represented by the polynomial \(16x^2 + 56x\). The width of the rectangle is given by the binomial \(2x+7\).

(a) Give a monomial expression in terms of \(x\) for the length of the rectangle. Show how you arrived at your answer.

(b) If the length of the rectangle is 80, what is the width of the rectangle? Explain your thinking.

Reasoning

7. These crazy polynomials keep acting like integers. We can factor integers to determine their factors. We can also do the same for polynomials.

(a) List all of the positive factors of the integer 12 by writing all possible positive integer products (such as \(12 = 3 \cdot 4\)).

(b) List all of the factors of \(2x^2 - 6x\) by also writing all possible products, such as \(2(x^2 - 3x)\).

8. Which of the following is not a factor of \(4x^2 + 12x\)?

(1) \(x + 3\)  
(2) \(x\)  
(3) \(3x\)  
(4) \(4\)
FACTORIZING BASED ON CONJUGATES
COMMON CORE ALGEBRA I

There are a number of different types of factoring techniques. But, each one of them boils down to reversing a product. We begin the lesson today by looking at products of conjugate binomials, or binomials of the form \(a+b\) and \(a-b\).

Exercise #1: Find each of the following products of conjugate pairs. See if you can work out a pattern.

(a) \((x+5)(x-5)\)
\[x^2-25\]
\[x^2-25\]

(b) \((x-2)(x+2)\)
\[x^2+2x-2x-4\]
\[x^2-4\]

(c) \((4x-1)(4x+1)\)
\[16x^2-1\]

(d) \((x+y)(x-y)\)
\[x^2-y^2\]
\[x^2-y^2\]

(e) \((2x+3)(2x-3)\)
\[4x^2-9\]
\[4x^2-9\]

(f) \((5x+2y)(5x-2y)\)
\[25x^2-4y^2\]
\[25x^2-4y^2\]

What we should see is that if we multiply conjugates, opposites always cancel and instead of getting our expected trinomial, we still get a binomial. Specifically.

MULTIPLYING CONJUGATE PAIRS
\[(a+b)(a-b)=a^2-b^2\]

Exercise #2: Use the pattern from Exercise #1 to quickly rewrite the following products.

(a) \((x+6)(x-6)\)
\[x^2-36\]
\[x^2-36\]

(b) \((5x+2)(5x-2)\)
\[25x^2-4\]
\[25x^2-4\]

(c) \((2x+7y)(2x-7y)\)
\[4x^2-49y^2\]
\[4x^2-49y^2\]

(d) \((4+x)(4-x)\)
\[16-x^2\]
\[16-x^2\]

(e) \((6+2y)(6-2y)\)
\[36-4y^2\]
\[36-4y^2\]

(f) \((10x-4y)(10x+4y)\)
\[100x^2-16y^2\]
\[100x^2-16y^2\]
We now should be able to reverse this multiplication in order to rewrite expressions that are the **difference of perfect squares** into products.

**Exercise #3:** Write each of the following first in the form \( a^2 - b^2 \) and then as equivalent products of conjugate pairs.

(a) \( x^2 - 81 \)
\[
(\sqrt{x} - 9)(\sqrt{x} + 9) = (x - 9)(x + 9)
\]

(b) \( 9x^2 - 4 \)
\[
(3x - 2)(3x + 2)
\]

(c) \( 25 - y^2 \)
\[
(5 - y)(5 + y)
\]

(d) \( 4x^2 - 81y^2 \)
\[
(2x - 9y)(2x + 9y)
\]

(e) \( 121x^2 - 1 \)
\[
(11x - 1)(11x + 1)
\]

(f) \( 1 - 4x^2 \)
\[
(1 - 2x)(1 + 2x)
\]

Never forget that when we factor, we are always rewriting an expression in a form that might look different, but it is ultimately still equivalent to the original.

**Exercise #4:** Let's take a look at the binomial \( x^2 - 9 \).

(a) Amelia believes that \( x^2 - 9 \) can be factored as \( (x+1)(x-9) \) while her friend Isabel believes that it is factored as \( (x-3)(x+3) \). Fill out the table below to develop evidence as to who is correct. Use technology on your calculator to help.

<table>
<thead>
<tr>
<th>x</th>
<th>( x^2 - 9 )</th>
<th>( (x+1)(x-9) )</th>
<th>( (x-3)(x+3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-9</td>
<td>-9</td>
<td>-9</td>
</tr>
<tr>
<td>1</td>
<td>-8</td>
<td>-16</td>
<td>-8</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>-21</td>
<td>-5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-24</td>
<td>0</td>
</tr>
</tbody>
</table>

Isabel is correct.

(b) By multiplying out their respective factors, show which of the two friends has the correct factorization. Use the Distributive Property Twice.

**Amelia:** \((x+1)(x-9)\)
\[
\begin{align*}
  x^2 - 9x + x - 9 & \\
  x^2 - 8x - 9 &
\end{align*}
\]

**Isabel:** \((x-3)(x+3)\)
\[
\begin{align*}
  x^2 - 3x + 3x - 9 & \\
  x^2 - 9 &
\end{align*}
\]
Name: __________________________________________  Date: __________________

FACTORIZING BASED ON CONJUGATE PAIRS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Use the fact that the product of conjugates follows the following pattern, \((a+b)(a-b)=a^2-b^2\), to quickly find the following products in standard form.

(a) \((x-5)(x+5)\)

\[x^2-25\]

(b) \((x+7)(x-7)\)

\[x^2-49\]

(c) \((2-x)(2+x)\)

\[-x^2\]

(d) \((3x+2)(3x-2)\)

\[9x^2-4\]

(e) \((4x+1)(4x-1)\)

\[16x^2-1\]

(f) \((2x+1)(2x-1)\)

\[4x^2-1\]

(g) \((5-4x)(5+4x)\)

\[25-16x^2\]

(h) \((x^2-2)(x^2+2)\)

\[x^4-4\]

(i) \((x^2+4)(x^2-4)\)

\[x^4-16\]

2. Write each of the following binomials as an equivalent product of conjugates.

(a) \(x^2-16\)

\[x^2-4^2\]

\((x+4)(x-4)\)

(b) \(x^2-100\)

\[x^2-10^2\]

\((x+10)(x-10)\)

(c) \(x^2-1\)

\[x^2-1^2\]

\((x+1)(x-1)\)

(d) \(x^2-25\)

\[x^2-5^2\]

\((x+5)(x-5)\)

(e) \(4-x^2\)

\[2^2-x^2\]

\((2-x)(2+x)\)

(f) \(9-x^2\)

\[3^2-x^2\]

\((3-x)(3+x)\)

(g) \(4x^2-1\)

\[2^2x^2-1^2\]

\((2x-1)(2x+1)\)

(h) \(16x^2-49\)

\[4^2x^2-7^2\]

\((4x-7)(4x+7)\)

(i) \(1-25x^2\)

\[1^2-5^2x^2\]

\((1-5x)(1+5x)\)

(j) \(x^2-9y^2\)

\[x^2-3^2y^2\]

\((x-3y)(x+3y)\)

(k) \(81-4t^2\)

\[9-2^2t^2\]

\((9-2t)(9+2t)\)

(l) \(x^4-36\)

\[(x^2-6)(x^2+6)\]
APPLICATIONS

3. A square is changed into a new rectangle by increasing its width by 2 inches and decreasing its length by 2 inches. Make sure to draw pictures to help you solve these problems!

(a) If the original square had a side length of 8 inches, find its area and the area of the new rectangle. How many square inches larger is the square’s area?

(b) If the original square had a side length of 20 inches, find its area and the area of the new rectangle. How many square inches larger is the square’s area?

(c) If the square had a side length of $x$ inches, show that its area will always be four square inches more than the area of the new rectangle.

REASONING

4. Consider the numerical expression $51^2 - 49^2$.

(a) Use your calculator to find the numerical value of this expression.

(b) Can you used facts about conjugate pairs to show why this difference should work out to be the answer from (a)?

5. Consider the following expression $(x + 2)(x - 2) - (x + 4)(x - 4)$.

(a) Using your calculator, determine the value of this expression for various values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$(x + 2)(x - 2) - (x + 4)(x - 4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

(b) Algebraically show that this product has a constant value (seen in (a)) regardless of the value of $x$. 
FACTORING TRINOMIALS
COMMON CORE ALGEBRA I

So far we have two factoring techniques: (1) Factoring out a g.c.f. and (2) Factoring based on conjugate pairs (factoring the difference of perfect squares). Today we will tackle the most difficult of the factoring techniques, and that is factoring trinomials. First, let's make sure we can multiply binomials.

Exercise #1: Write each of the following products in equivalent trinomial form.

(a) \((x+5)(x+3)\)
\[x^2 + 8x + 15\]
\[x^2 + 5x + 3x + 15\]

(b) \((x-2)(x+7)\)
\[x^2 + 5x - 14\]
\[x^2 + 7x - 2x - 14\]

(c) \((x-5)(x-10)\)
\[x^2 - 15x + 50\]
\[x^2 - 10x - 5x + 50\]

Try a few that are a bit more difficult. It is critical that you are fluent with this type of multiplication before we try to reverse the process.

Exercise #2: Write each of the following products in equivalent trinomial form.

(a) \((2x-3)(5x+1)\)
\[10x^2 + 2x - 15x - 3\]
\[10x^2 - 13x - 3\]

(b) \((6x+7)(x+2)\)
\[6x^2 + 12x + 7x + 14\]
\[6x^2 + 19x + 14\]

(c) \((4x-1)(2x-5)\)
\[8x^2 - 8x - 2x + 5\]
\[8x^2 - 10x + 5\]

Now, we need to reverse this process to take a trinomial and write it as the product of the binomials. There is truly only one fail proof method for this type of factoring and that is GUESS AND CHECK. This method often frustrates students, but look at it as a puzzle and make “smart” guesses and quick checks.

Exercise #3: Consider the trinomial \(2x^2 + 13x + 20\). We want to write it as the product of two binomials, in other words, reverse what we did in Exercises #1 and #2.

(a) Fill in the missing blanks with the only pair that makes sense (that is a smart guess).

\[____ + \quad)(____ + \quad)\]

(b) Why does it make sense that both of the binomials will be addition (as shown in (a))?

(c) List pairs of factors that can produce a 20.

"AC" method: \[A\times C\] + \([B\times C] = 20 \quad a=2 \quad b=13 \quad C=20\]

\[a \times C = 40\]

5 \times 8 \quad \text{what two numbers multiply to 40 and add to 13}

\[19 + 0\]
\[2 + 20\]
\[5 + 8\]

Try various pairs from (c), checking only that the linear terms will sum to 13x until you find the correct factorization.
There is absolutely no substitution for rote practice with factoring trinomials. To be able to do this critical skill, you must be smart with your guesses and patient with your checks. You will find the correct answer, but you must be willing to guess incorrectly multiple times.

**Exercise #4:** Write each of the following trinomials in equivalent factored form. Show your checks and don’t worry if your first guess isn’t correct. You will get better at these! Note for yourself which ones seemed easier and which were harder.

(a) $3x^2 + 5x - 2$

(b) $x^2 - 7x + 10$

(c) $x^2 + 10x + 21$

(d) $10x^2 + 9x + 2$

(e) $8x^2 - 18x + 9$

(f) $2x^2 + 5x + 2$

(g) $x^2 - 8x + 12$

(h) $7x^2 - 26x - 8$

(i) $5x^2 - 5x + 2$

(j) $10x^2 - 20x + 10$

(k) $x^2 - 5x + 4$

(l) $10x^2 - 20x + 20$

(m) $x^2 - 2x + 3$

(n) $x^2 - 6x + 9$

(o) $x^2 - 6x + 1$

(p) $x^2 - 6x + 1$

(q) $x^2 - 6x + 1$

(r) $x^2 - 6x + 1$

(s) $x^2 - 6x + 1$

(t) $x^2 - 6x + 1$

(u) $x^2 - 6x + 1$

(v) $x^2 - 6x + 1$

(w) $x^2 - 6x + 1$

(x) $x^2 - 6x + 1$

(y) $x^2 - 6x + 1$

(z) $x^2 - 6x + 1$

**COMMON CORE ALGEBRA 1, UNIT #7 - POLYNOMIALS - LESSON #5**

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FACTORYING TRINOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following products is equivalent to the trinomial $x^2 - 5x - 24$?

   (1) $(x-12)(x+2)$  
   (2) $(x+12)(x-2)$  
   (3) $(x-8)(x+3)$  
   (4) $(x+8)(x-3)$

2. Written in factored form, the trinomial $2x^2 + 15x + 28$ can be expressed equivalently as

   (1) $(2x+7)(x+4)$  
   (2) $(2x+4)(x+7)$  
   (3) $(2x+2)(x+14)$  
   (4) $(x+7)(2x+7)$

3. (Easier) Write each of the following trinomials in equivalent factored form. Keep at it!!! Use extra scrap paper for extra guesses.

   (a) $x^2 + 10x + 16$  
   (b) $x^2 + 12x + 35$  
   (c) $x^2 + 12x + 36$

   (d) $x^2 - 5x + 6$  
   (e) $x^2 - 11x + 28$  
   (f) $x^2 - 12x + 20$

   (g) $x^2 - 3x - 18$  
   (h) $x^2 + 3x - 40$  
   (i) $x^2 - 10x - 24$

   (j) $x^2 - 8x + 15$  
   (k) $x^2 + 30x + 200$  
   (l) $x^2 + 8x - 9$
4. (Medium) Write each of the following trinomials in equivalent factored form. Keep at it!!! Use extra scrap paper for extra guesses.

(a) $2x^2 + 13x + 21$
(b) $5x^2 - 21x + 4$
(c) $3x^2 + 16x - 12$
(d) $7x^4 + 11x^2 - 6$
(e) $2x^2 - x - 10$

5. (Hardest) Write each of the following trinomials in equivalent factored form. Keep at it!!! These might require quite a few tries. You will get better at them as you practice. Use extra scrap paper for extra guesses.

(a) $4x^2 + 27x + 18$
(b) $6x^2 + 5x - 4$
(c) $12x^2 - 31x + 20$

APPLICATIONS

6. A rectangle has dimensions as shown below in terms of an unknown variable, $x$.

(a) Find a binomial expression for the width of the rectangle in terms of $x$. Justify your answer based on the expressions for the rectangle’s length and area.

Length = $x + 4$
Area = $2x^2 + 9x + 4$

(b) If the width of the rectangle is 21 inches, what is the length and the area? Use appropriate units and explain how you found your answer.
MORE WORK FACTORING TRINOMIALS
COMMON CORE ALGEBRA I

Factoring trinomials, which we first practiced in the last lesson, is a trying experience. All algebra students must learn how to do this procedure because of its immense number of practical applications. We will eventually see these applications, but for now, we need to get more practice factoring these trinomials. We begin by looking at a process known as complete factoring.

Exercise #1: Consider the trinomial $4x^2 + 20x + 24$.

(a) Write this trinomial as an equivalent expression involving the product of its term’s gcf and another trinomial.

(b) Factor this additional trinomial to express the original in completely factored form.

Whenever we factor, we should always look to see if a greatest common factor exists that can be “factored out” to begin the problem. This will always make any subsequent factoring easier.

Exercise #2: Rewrite each of the following trinomials in completely factored form.

(a) $10x^2 + 15x - 10$

\[5(2x^2 + 3x - 2)\]

\[5\left(2x^2 + 4x - 2\right)\]

\[5(2x(x+2) - 1(x+2))\]

\[5(2x-1)(x+2)\]

(b) $3x^2 - 21x^2 + 36x$

\[3x(x^2 - 7x + 12)\]

\[3x(x-3)(x-4)\]

(c) $7x^2 + 21x - 70$

\[7(x^2 + 3x - 10)\]

\[7(x + 5)(x-2)\]

(d) $6x^2 - 2x - 4$

\[2(3x^2 - x - 2)\]

\[2(3x^2 - 3x + 2x - 2)\]

\[2(3x(x - 1) + 2(x - 1))\]

\[2(3x+2)(x-1)\]
Complete factoring can also involve factoring the difference of perfect squares. Try the next exercise to see how this works.

Exercise #3: Write each of the following binomials in completely factored form.

(a) \(2x^2 - 18\)
\[2(x^2 - 9)\]
\[2(x + 3)(x - 3)\]

(b) \(5x^3 - 20x\)
\[5x(x^2 - 4)\]
\[5x(x + 2)(x - 2)\]

(c) \(12x^2 - 3\)
\[3(4x^2 - 1)\]
\[3(2x - 1)(2x + 1)\]

(d) \(54x^2 - 24\)
\[6(9x^2 - 4)\]
\[6(3x - 2)(3x + 2)\]

If you understand factoring as breaking an expression into an equivalent product, then essentially you can always check to see if you have factored correctly. Complete factoring actually leads to a nice way to eliminate some guesses from trinomial guess and check methods.

Exercise #4: Consider the trinomial \(2x^2 + 11x + 12\).

(a) Do the three terms of this trinomial have a gcf other than 1?

(b) Why would the guesses \((2x + 2)(x + 6)\), \((2x + 4)(x + 3)\), and \((2x + 12)(x + 1)\) not make sense given your answer to (a)?

(c) Fill in the statement:

If a trinomial does not have a gcf, then

\[\text{_________ of its ___________ factors will}
\[\text{have a gcf.}\]

(d) Factor this trinomial by limiting your guesses.

Exercise #5: Use the Smart Guessing Tip from the last problem to factor \(4x^2 - 21x - 18\).
MORE WORK FACTORING TRINOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Rewrite each of the following trinomials in completely factored form.

(a) \(2x^2 + 20x + 42\)
   \[2(x^2 + 10x + 21)\]
   \[2(x + 3)(x + 7)\]

(b) \(6x^2 + 33x + 15\)
   \[3(2x^2 + 11x + 5)\]
   \[3(2x + 5x + 5)\]
   \[3(2x + 1)(x + 5)\]

(c) \(5x^2 - 10x - 40\)
   \[5(x^2 - 2x - 8)\]
   \[5(x - 4)(x + 2)\]

(d) \(30x^2 + 20x - 10\)
   \[10(3x^2 + 2x - 1)\]
   \[10(3x^2 - x + 3x - 1)\]
   \[10(3x - 1)(x + 1)\]

(e) \(x^3 + 7x^2 + 10x\)
   \[x(x^2 + 7x + 10)\]
   \[x(x + 5)(x + 2)\]

(f) \(4x^3 + 10x^2 - 24x\)
   \[2x(2x^2 + 5x - 12)\]
   \[2x(2x^2 - 3x - 3x + 12)\]
   \[2x(x + 3)(x - 4)\]

(g) \(5x^2 - 45\)
   \[5(x^2 - 9)\]
   \[5(x + 3)(x - 3)\]

(h) \(2x^3 - 2x\)
   \[2x(x - 1)\]
   \[2x(x + 1)(x - 1)\]

(i) \(36 - 4x^2\)
   \[4(9 - x^2)\]
   \[4(3 - x)(3 + x)\]

(j) \(20x^2 - 125\)
   \[5(4x^2 - 25)\]
   \[5(2x^2 - 5^2)\]
   \[5(2x + 5)(2x - 5)\]

2. Which of the following is not a factor of \(4x^3 + 12x^2 - 72x\)? Show work that justifies your choice.

(1) \(x + 9\)
(2) \(4x\)
(3) \(x - 3\)
(4) \(x + 6\)

\[4x(x^2 + 3x - 18)\]
\[4x(x + 6)(x - 3)\]
3. Which of the following is the missing factor in the product $2(x-1)( \ ? \ )$ if it is equivalent to the trinomial $2x^2+10x-12$? 

(1) $x+12$  
(2) $x+6$  
(3) $x+3$  
(4) $x-5$

4. Use the Smart Guessing Tip from Exercise #4 to help factor the following challenging trinomials. Note that they do not have a greatest common factor.

(a) $4x^2+19x+12$ 
\[
\begin{array}{c}
4x^2+19x+12 \\
\downarrow \quad \downarrow \\
16x \\
19x \\
12 \\
\end{array}
\]

(b) $6x^2+7x-24$ 
\[
\begin{array}{c}
6x^2+7x-24 \\
\downarrow \quad \downarrow \\
16x \\
9x \\
-14y \\
\end{array}
\]

5. Consider the cubic trinomial $x^3+8x^2+7x$.

(a) Write this trinomial as an equivalent product in completely factored form.

\[x(x^2+8x+7)\]

(b) How can the original trinomial and your answer to (b) help you determine the value of $(10)(17)(11)$ without a calculator? What is the value?

6. Use the complete factorization of $2x^3+8x^2+8x$ to determine the value of the product $(20)(12)^2$. Explain your reasoning.

\[2x(x^2+4x+4)\]
\[2x(x+2)(x+2)\]